Multiphase Wattmeters Based on the Magnetoresistance Effect of Semiconductors[*]

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INTRODUCTION

IN previous publications, multiphase wattmeters have been described, which are based on the Hall-effect of semiconductors.[1-3] With these wattmeters, the semiconductor compounds indium-antimonide and indium-arsenide were used. The carriers in these compounds show much greater mobilities than in germanium or silicon. Hence, the Hall-effect is also much greater. As a consequence of the Hall-effect, the resistance of a semiconductor disk, *e.g.*, of rectangular shape, is greater in the presence of a transverse magnetic field than without such a field. This magnetoresistance effect is often given the name of C. F. Gauss. It is very much dependent on the shape of the probe, as is seen from Fig. 1, taken from Welker.[4] It is most pronounced for a circular disk with one central contact and with the other contact in the shape of a circumferential ring. As is seen from Fig. 1, the resistance of such a disk of indium-antimonide at a transverse flux density of 10 kilogauss may be about 18 times as large as with zero flux density.

SINGLE-PHASE WATTMETERS FOR MEASURING REAL POWER

From the curves of Fig. 1, we may conclude that the increase of resistance is about proportional to the square of the corresponding increment of flux density at small flux density values (*e.g.*, 1 kilogauss). At large flux density values (*e.g.*, 6 kilogauss) the increase of resistance is approximately proportional to the first power of the corresponding increment of flux density. This fact may be expressed mathematically as follows:

$$R - R_B = \left(\frac{dR}{dB}\right)_B \Delta B + \frac{1}{2}\left(\frac{d^2R}{dB^2}\right)_B (\Delta B)^2$$
$$+ \frac{1}{6}\left(\frac{d^3R}{dB^3}\right)_B (\Delta B)^3 + \cdots . \tag{1}$$

[1] W. Hartel, "Anwendung der Hallgeneratoren," *Siemens Z.*, vol. 28, pp. 376–384; September, 1954.
[2] S. F. Sun, "Power Measurement and Power Regulation in Multiphase Networks by Means of Semiconductors," Ph.D. thesis, Swiss Federal Institute of Technology, Zurich, Switzerland, Nr. 2518; 1955.
[3] M. J. O. Strutt and S. F. Sun, "Leistungsmessung und Leistungsregulierung in Mehrphasennetzen mittels Halbleitern," *Arch. Elektrotech.*, vol. 42, pp. 155–164; Nr. 3, 1955.
[4] H. Welker, "Neuere Untersuchungen der Halbleitereigenschaften von III–V-Verbindungen," *Scientia Electrica*, vol. 1, pp. 152–164; November, 1954.

The differential coefficients refer to the curve, representing the resistance R as dependent on flux density B. Their values are taken at the flux density B under consideration. At small flux densities B, the first differential coefficient, multiplied by $\Delta B/R_B$ is small with respect to unity; the second differential coefficient, multiplied by $(\Delta B)^2/R_B$ is much larger, whereas the third differential coefficient, multiplied by $(\Delta B)^3/R_B$ is again much smaller. At large flux densities B, the first differential coefficient, multiplied by $\Delta B/R_B$ is much larger than the second differential coefficient, multiplied by $(\Delta B)^2/R_B$ and than the third differential coefficient, multiplied by $(\Delta B)^3/R_B$. Hence, at large flux densities we may write with good approximation:

$$R - R_B = R_W = \left(\frac{dR}{dB}\right)_B \Delta B. \tag{2}$$

With

$$\Delta B = B_{max} \sin \omega t, \tag{3}$$

and $B_{max} \sin \omega t$ proportional to the phase voltage $E_{max} \sin \omega t$, we obtain

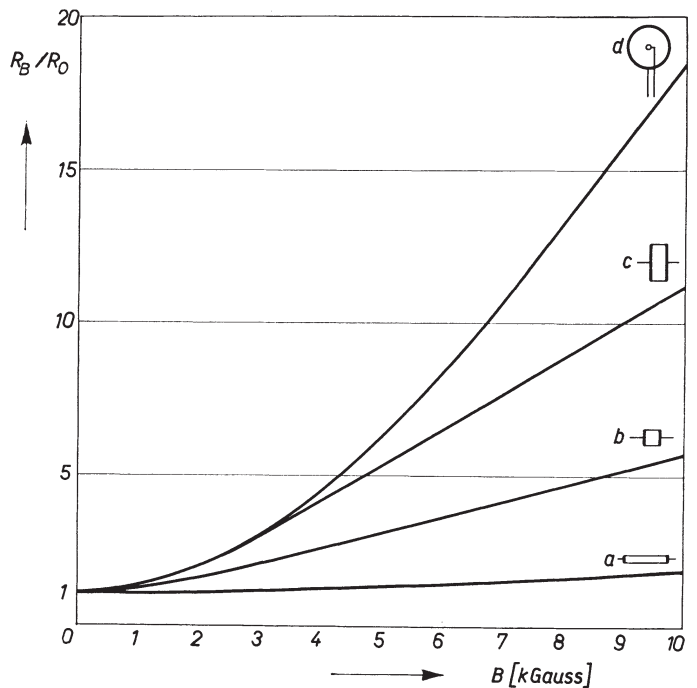$$R_W = k_E E_{max} \sin \omega t. \tag{4a}$$

Fig. 1—Vertical scale: ratio of resistance R_B at a transverse magnetic flux density B to the resistance R_0 at zero flux density for disks made of indium-antimonide of different shapes as indicated.

The current i_s through the semiconductor is assumed to be proportional to the phase current $I_{max} \sin (\omega t + \varphi)$. Then,

$$i_s = k_I I_{max} \sin (\omega t + \varphi). \qquad (4b)$$

We shall presently discuss the circuits for which these relationships hold. The voltage e_s across the semiconductor contacts is

$$
\begin{aligned}
e_s &= i_s(R_W + R_B) \\
&= k_E k_I E_{max} I_{max} \tfrac{1}{2} \cos \varphi \\
&\quad - k_E k_I E_{max} I_{max} \tfrac{1}{2} \cos (2\omega t + \varphi) \\
&\quad + k_I I_{max} R_B \sin (\omega t + \varphi). \qquad (5)
\end{aligned}
$$

It appears that the voltage e_s contains three components: one dc component e_{s0}, one ac component with an angular frequency 2ω and a third component with an angular frequency ω.

A circuit, for which the relationships (4a) and (4b) hold, is shown in Fig. 2. The coil S is wound on a yoke of suitable material, *e.g.*, permalloy, and has the inductance L. The resistance r_B satisfies the condition $r_B \gg L$. In this case, the current through S is proportional to the phase voltage $E_{max} \sin \omega t$. The ac magnetic flux density ΔB is then also proportional to this phase voltage. A permanent magnet M, inserted into the said yoke, causes a constant flux density B in the air gap, upon which ΔB is superimposed. The air gap contains the semiconductor element, which may, *e.g.*, have the shape of a circular disk with a central and a circumferential ring contact. The current through this disk is made proportional to the phase current $I_{max} \sin (\omega t + \varphi)$ by the use of suitable resistances r and r_I. The latter resistance r_I is large with respect to the resistance of the disk. Hereby the current through the disk is independent of its resistance. This is important in view of the temperature dependence of the disk's resistance. It is, of course, possible to apply a suitable voltage transformer instead of r_B, with a suitable resistance r_B in its secondary circuit, so as to make sure that the current through S is proportional to the phase voltage. The important conditions to be satisfied by the circuit of Fig. 2 are: 1) The ac magnetic flux density ΔB must be proportional to the phase voltage with respect to magnitude and phase angle. 2) The current i_s must be proportional to the phase current with respect to magnitude and phase angle. 3) The current i_s must be independent of the resistance of the semiconductor disk.

If these conditions are satisfied, (5) shows that the dc component e_{s0} of the voltage e_s across the disk electrodes is proportional to the real power, absorbed by the impedance Z. The power absorbed by the disk and by the resistances r and r_I has been neglected. As a fourth condition, this latter power, hence, must be small compared with the power absorbed by Z. An analogous condition also holds for wattmeter circuits of the usual type.

The dc voltage e_{s0} causes a dc in the circuit, consisting of r_I and of r. This dc causes a voltage drop across the disk's resistance R_B, which lowers the resulting dc voltage at the disk's electrodes. The dc may be prevented by the insertion of a condenser C into the circuit. At an angular frequency ω, the impedance of this condenser must be small with respect to the resistance r_I, so as not to cause a phase angle of the current i_s with respect to the phase current. This condition may be formulated thus: $r_I \omega C \gg 1$. This fifth condition must be added to those already mentioned. If the resistance r_I is sufficiently large in comparison to the resistance R_B of the disk, the said dc will be small and practically no voltage drop will be caused at the disk's electrodes. In this case, the condenser C may be omitted.

Another circuit, suitable for measuring the real power in a single phase network, is shown in Fig. 3. In this circuit, the ac magnetic flux density ΔB is proportional to the phase current $I_{max} \sin (\omega t + \varphi)$, while the current through the semiconductor element is proportional to the phase voltage $E_{max} \sin \omega t$;

$$R_W = k_I' I_{max} \sin (\omega t + \varphi)$$
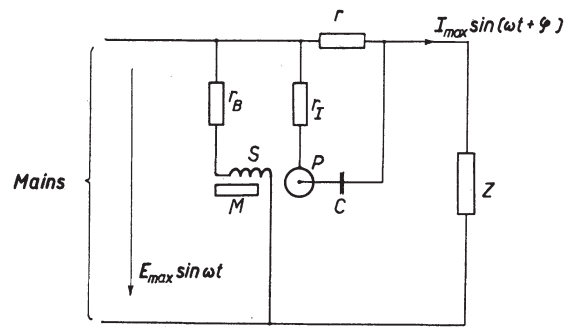
$$i_s = k_E' E_{max} \sin (\omega t).$$

Fig. 2—Wattmeter circuit using a semiconductor disk P, suitable for the determination of the real power, absorbed in the impedance Z. The disk is assumed to be of circular shape with a central and a circumferential contact (see upper curve of Fig. 1).
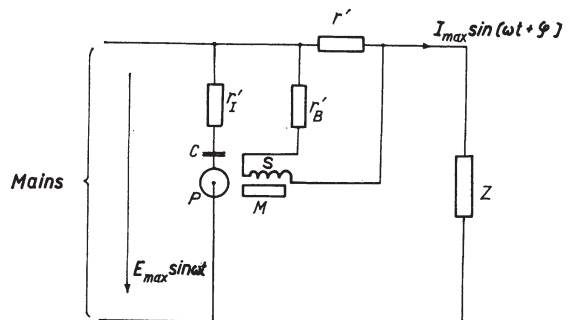
Fig. 3—Wattmeter circuit serving the same aim as that of Fig. 2.

The resulting voltage e_s' across the semiconductor electrodes is hence:

$$e_s' = k_E' k_I' E_{max} I_{max} \tfrac{1}{2} \cos \varphi$$
$$- k_E' k_I' E_{max} I_{max} \tfrac{1}{2} \cos (2\omega t + \varphi)$$
$$+ k_E' E_{max} R_B \sin \omega t.$$

The dc voltage component, corresponding to the first term of this equation is proportional to the real power consumed in the impedance Z of Fig. 3. The conditions to be satisfied by the circuit of Fig. 3 are: 1) the ac magnetic flux density ΔB must be proportional to the phase current with respect to amplitude and phase angle. 2) The current i_s through the semiconductor must be proportional to the phase voltage with respect to amplitude and phase angle. 3) The current i_s must be independent of the resistance R_B of the semiconductor. 4) The power, absorbed by the coil S and by the resistances r_B' and r', must be small in comparison to the power absorbed in Z. 5) The condenser C must satisfy the condition $r_I'\omega C \gg 1$. These conditions may be satisfied by a suitable choice of the resistances r_I', r_B' and r'. Of course, suitable current and voltage transformers may also be applied in the present case.

SINGLE-PHASE WATTMETERS MEASURING REACTIVE POWER AND APPARENT POWER

If the resistance r_B is omitted in Fig. 2, the ac magnetic flux density ΔB is proportional to the integral of the phase voltage with respect to time. This holds also for the ac resistance R_W. Hence,

$$R_W'' = k_E'' E_{max} \sin \left(\omega t + \frac{\pi}{2} \right). \tag{6}$$

Eq. (5), in this case, has to be replaced by

$$e_s'' = i_s(R_W'' + R_B)$$
$$= k_E'' k_I E_{max} I_{max} \tfrac{1}{2} \cos \left(\varphi - \frac{\pi}{2} \right)$$
$$- k_E'' k_I E_{max} I_{max} \tfrac{1}{2} \cos \left(2\omega t + \varphi + \frac{\pi}{2} \right)$$
$$+ k_I I_{max} R_B \sin (\omega t + \varphi). \tag{7}$$

The dc component e_{s0} of the voltage e_s'' across the semiconductor element is now proportional to the reactive power flowing into the impedance Z. In order to assure a sufficient accuracy of this proportionality, conditions 2 through 4 of Fig. 2 must be satisfied in the present case. Furthermore, (6) must hold with sufficient accuracy.

We may also apply current and voltage transformers of suitable construction in the present case.

As to the apparent power, it is seen that the amplitude of the voltage component of angular frequency 2ω in (7) and in (5) is proportional to the apparent power flowing into the impedance Z. Hence, by application of a frequency-selective voltmeter for the measurement of this amplitude, we may determine this apparent power.

MULTIPHASE WATTMETERS

In multiphase networks with ground conductor, the above circuits of Figs. 2 and 3 may be applied to each phase. The dc voltages arising at the semiconductor terminals may then be added. The ensuing dc voltage is proportional to the real or to the reactive power of the multiphase system. We shall now discuss this more fully, using a three-phase system with ground conductor as an example.

Fig. 4 shows a wattmeter for measuring real power. The three semiconductor disks P_1, P_2 and P_3 are fed by the three voltage transformers T_1, T_2 and T_3. In each circuit, a series resistance r and a condenser C are inserted. The resistance r is large compared with the resistance R_B of one disk and satisfies the condition $\omega r C \gg 1$. Hereby, the current through each disk is independent of its resistance R_B and, furthermore, is proportional to the phase voltage of the system with respect to magnitude and phase angle. The transverse magnetic flux density is proportional to the phase current. The constant flux density B is caused by the permanent magnet M. The circuits of Fig. 4 are for each phase equivalent to the circuit of Fig. 3. The conditions, stated in connection with the latter figure, hold also for Fig. 4. The resulting dc voltage E_0 of the three semiconductor disks is measured by means of a voltmeter of high internal resistance. This high resistance prevents the existence of a shunt dc current path in parallel to the disk. Therefore, the voltage E_0 is truly proportional to the real power of the system.

In order to measure the reactive power in a three-phase system with ground conductor, the circuit of Fig. 5 may be applied. The ac magnetic flux density is now caused by the phase voltage, according to (6). The semiconductor disks are fed by current transformers. Thus, (7) holds. The resulting dc voltage E_0 of the three disks is proportional to the reactive power of the system. The conditions, which have to be satisfied, are those stated above in connection with (6) and (7).

The condensers C may be omitted, if the resistances r are sufficiently large in comparison to the resistance R_B of each disk. In this case, the dc currents flowing through r are practically negligible.

COMPENSATION OF TEMPERATURE EFFECTS

With disks of indium-antimonide, the resistances R_B and R_W are strongly dependent on temperature. With indium-arsenide, this temperature-dependence is less
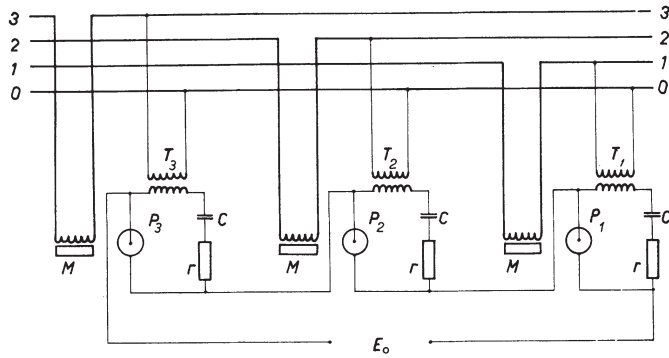
Fig. 4—Wattmeter circuit using three semiconductor disks P_1, P_2, and P_3 suitable for the measurement of the real power, transported by a three-phase power system. The disks are fed by the voltage transformers T_1, T_2, and T_3. The dc voltage E_0 is proportional to the real power of the system.
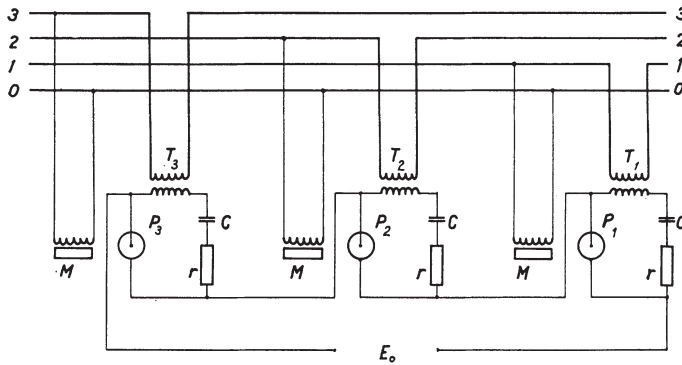

Fig. 5—Wattmeter circuit suitable for the measurement of reactive power in a three-phase power system. The three semiconductor disks P_1, P_2, and P_3 are fed by the three current transformers T_1, T_2, and T_3. The dc voltage E_0 is proportional to the reactive power of the system.

pronounced. However, in both cases, special circuits must be applied in order to obtain sufficient accuracy of the wattmeters at different temperatures.

Fig. 6 (next page) shows the resistance of a disk of rectangular shape made of indium-antimonide as dependent on temperature (degrees centigrade) at several values of the transverse magnetic flux density B. These curves show that in the temperature range, marked by the vertical lines A and B, this temperature dependence may be approximately described by an exponential function. The curves of Fig. 7 have been obtained from those of Fig. 6. These curves show that R_B as well as R_W change markedly in the temperature range between -20 and $+60°C$. Fig. 8 shows the resistance of a disk made of indium-arsenide, at zero-transverse magnetic flux density, as dependent on temperature. Obviously, this temperature-dependence is much less marked than with indium-antimonide.

The compensation of these temperature effects starts from (4). A change of temperature causes a change of k_E, whereas k_I remains constant, as i_s does not depend

on the disk's resistance R_B. The aim of the temperature compensation under consideration is to keep the *product* $k_E k_I$ independent of temperature within a predetermined temperature-range. Hence, k_I must be made temperature-dependent too. In order to achieve this, the current path through the semiconductor disk in Figs. 2 and 3 must be altered. At rising temperature, the ac magnetic flux density ΔB being constant, the value of R_W decreases according to Figs. 6 and 7. Hence, k_E also decreases. The value of k_I must increase in order to keep the product $k_I k_E$ constant. This is achieved by the circuit of Fig. 9. A resistance r_T is inserted in series with the disk (resistance R_B). A further resistance r_0 is connected in parallel to r_T and R_B. The resistance r_T is large compared with R_B, and r_0 is of the same order of magnitude as R_B. The resistance r_I is also large in comparison to R_B as was already stated in connection with Fig. 2.

From Fig. 6 it is seen that the temperature dependence of R_W [see (2)] is not very different from that of R_B at a definite transverse flux density B. This may be inferred from the almost parallel course of the curves at different values of B. The resistance r_T is made of the same material as the disk under consideration. The condition $r_T \gg R_B$ is satisfied. Hence, the current i_s is almost exclusively determined by r_T. The resistance r_0 is only slightly dependent on temperature and satisfies the condition $r_0 \ll r_T$. This entails that $i_0 \gg i_s$. The current i is almost exclusively determined by r_I and $i_0 \approx i$. The current i_s is thus approximately given by the equation $i_s \approx i r_0/r_T$. As i is independent of temperature (r_I being so), the quotient r_0/r_T determines the temperature dependence of i_s. This current, hence, rises with rising temperature proportional to r_0/r_T. The temperature dependence of R_W and hence of k_E is approximately the same as that of R_B. Hence, the temperature dependence of k_I, by the circuit of Fig. 9, becomes approximately the same as that of $1/R_B$. The product $k_E k_I$ may thus be made approximately independent of temperature as desired. This procedure is analogous to the one which was described previously[2,3] in connection with the Hall-effect wattmeters.

If a disk of indium-arsenide is used (see Fig. 8), it is possible to obtain a product $k_E k_I$, which is very nearly independent of temperature by application of the same procedure as described above. These compensating procedures may be carried out in a considerable range of temperature, e.g., between -20 and $+60°C$.

The current i_s may heat the disk and thereby cause an additional temperature dependence of k_E. The resistance r_T may also be heated. Precautions must be taken in order to avoid upsetting the above temperature compensation by these heating effects. The maximum current i_s should be so small as to cause no undue heating of the disk and of r_T.
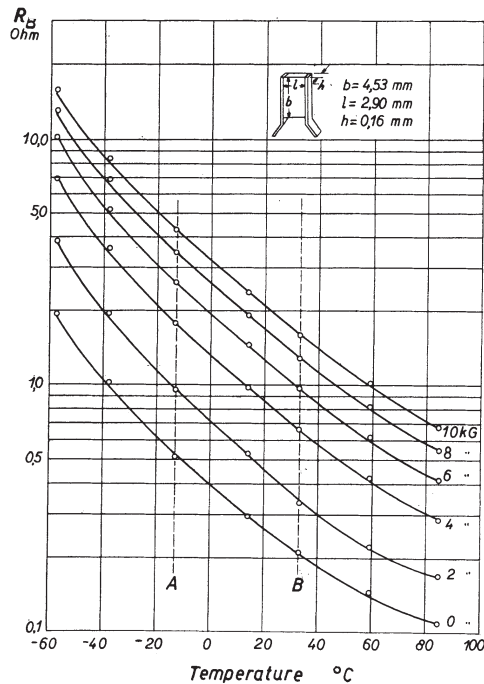
Fig. 6—Vertical scale: resistance R_B in ohms of an indium-antimonide disk of rectangular shape as indicated, as dependent on temperature (horizontal scale in degrees centigrade) at different values of transverse magnetic flux density B expressed in kilogauss. Between the vertical lines A and B, the curves are nearly exponential.
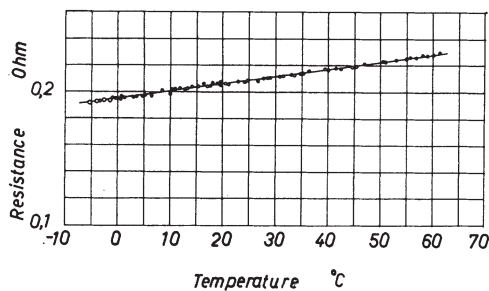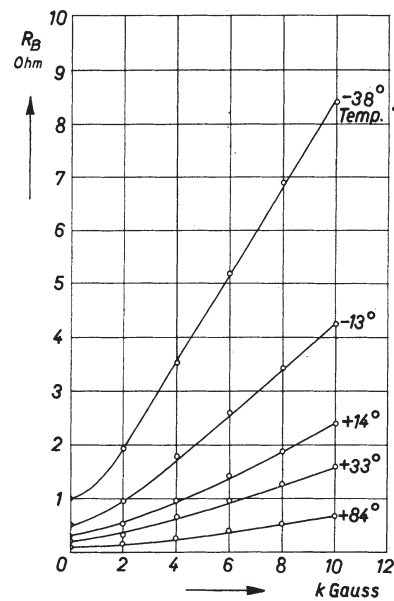

Fig. 7—These curves are taken from Fig. 6. Vertical scale: as in Fig. 6. Horizontal scale: transverse magnetic flux density B in kilogauss. Curves at different values of temperature in degrees centigrade.
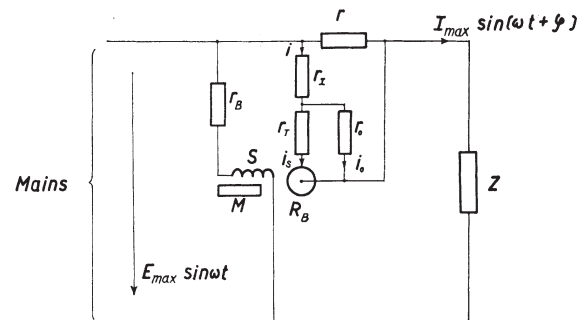

Fig. 8—Resistance of a disk of rectangular shape, made of indium-arsenide as dependent on temperature at zero magnetic flux density.

Fig. 9—Circuit suitable for the compensation of the temperature effects.

Order of Magnitude of the DC Voltages at the Terminals of the Semiconductor Disks

From Fig. 7, it may be seen that for this disk the value of R_W at $\Delta B = 2$ kilogauss and at $B = 6$ kilogauss at room temperature is about 0.4 ohm. If we use a circular disk with contacts at the center and at the circumference, the value of R_W may be much larger. We shall assume R_W to be 0.5 ohm at $\Delta B = 2$ kilogauss. This value of $\Delta B = 2$ kilogauss at $B = 6$ kilogauss represents about the limit of the linear range with the curves of Fig. 7.

The current i_s through the disk must be chosen such that no undue heating occurs. With the disk of Fig. 7, the maximum permissible value of i_s may be estimated at about 0.2 ampere. In this case, we obtain a dc voltage at the disk's terminals of $\frac{1}{2} \times 0.5 \times 0.2$ volt $= 50$ mv. This would be the highest dc voltage to be expected in

the present case. It might have any value between zero and 50 mv.

For the purposes of regulation in electrical power systems, it is very useful that the power indication is by a dc voltage. This voltage may be used for regulation. In so doing, we should bear in mind that not the entire available dc power at the disk's terminals may be used, but only a small fraction of this available dc power. According to the above rating, the maximum available dc power at a voltage of 50 mv and an internal resistance R_B of about 1 ohm would be about 600 μw. Of this power, only about one per cent may be used. If a larger fraction would be taken, the temperature dependence of R_B might influence the dc output voltage and might falsify the relation between this voltage and the power to be measured. This reasoning is equivalent to the one

put forward in the case of Hall-effect wattmeters.[2,3] Hence, only a few microwatts of dc power are available. This small power may, however, be amplified a few thousand times using a suitable magnetic amplifier. This has also been discussed previously[2,3] and a suitable amplifier has been constructed. Using this, a few milliwatts dc power may easily be obtained and this would be sufficient for regulation purposes in most cases.

FREQUENCY DEPENDENCE OF HIGH-FREQUENCY WATTMETERS

In many practical cases it is desirable to measure real and reactive power at frequencies higher than 50 or 60 cps. The wattmeters, described above, are suitable for these purposes with certain restrictions. In the first place, the dependence of R_B and of R_W on the frequency of the current through the disk and of the transverse magnetic flux density must be known.

Measurements, effected with disks of similar dimensions as shown in Fig. 6, have shown that R_B is hardly dependent on frequency up to about 10 Mc. At higher frequencies, R_B does depend on frequency. At 300 Mc the value of R_B dropped to about one half, and at 600 Mc to about one third of its value at zero frequency.[5]

Later unpublished measurements in these laboratories have shown that the frequency dependence of R_B and R_W is much less with circular disks having a central and a circumferential electrode. It may be stated that magnetoresistance wattmeters may now be built, functioning fautlesssly up to 10 Mc according to the circuits described above. It is probable that this functioning may be extended up to 300 Mc with proper precautions.

The semiconductor is exposed to a transverse magnetic ac flux. Especially in the case of a circular disk (upper curve of Fig. 1), a short circuit conducting ring is present in this alternating magnetic field, causing eddy current losses. These may be decreased by a subdivision of the contact ring, as shown in Fig. 10(b). The ends of the half-ring contacts are interconnected, as shown in Fig. 10. Further subdivision, creating four, six, eight etc. parts of the contact ring, is also possible. Hereby, a further reduction of eddy current losses is possible. With a rectangular disk, as shown in Fig. 10(a), a subdivision
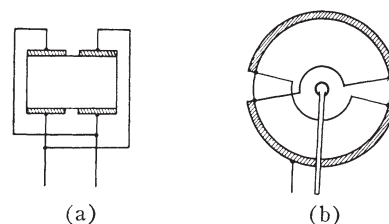

Fig. 10—Subdivision of contacts of semiconductor disks.

of the contacts also reduces the eddy current losses.[6] The connecting leads should preferably lie in a plane, which is parallel to the magnetic flux.

COMPARISON WITH HALL-EFFECT WATTMETERS

In comparing the magnetoresistance wattmeters with the Hall-effect wattmeters,[1-3] the following points may be made.

A disk, suitable for a Hall-effect wattmeter, has four or at least three contact terminals, whereas with the disks used here, only two terminals are required. This fact already entails more intricate circuits in the case of Hall-effect wattmeters than with magnetoresistance wattmeters.

In preparing the disks of Hall-effect wattmeters, four or at least three contacts must be made by some soldering process. With the present disks, only two contacts have been soldered. With four contacts, special conditions of symmetry must be satisfied. These also are not necessary with the present disks.

At high frequencies, special grounding precautions must be taken in the wattmeter circuit (connections to the chassis). These connections are much easier to effect with two than with three or four disk contacts.

The magnitudes of the resulting dc voltages are about equal with both types of wattmeters.

The temperature compensation is considered to be slightly simpler with the magnetoresistance wattmeters.

[5] P. Ramer, M. J. O. Strutt, and F. K. von Willisen, "Messungen des Gausseffektes verschiedener Halbleiter bei 10, 300 und 600 MHZ," *Arch. Elekt. Übertragung*, vol. 11, pp. 1–7; January, 1957.

[6] M. J. O. Strutt, "Einrichtung zur messtechnischen Ausnutzung des Gauss-Effektes mit Hilfe eines einem Magnetfeld ausgesetzten Gauss-Generators," Deutsches Bundes-Patent 1.000.924, im Besitze der A. G. Brown, Boveri & Cie., Baden, Switzerland; July 2, 1955.